Designing teaching modules on the history and philosophy of mathematics

– a report from an in-progress study in Danish high schools

Disposition

- Mathematics program for Danish high schools
- The KOM-report: mathematical competencies and ‘overview and judgment’
- Question
- Designing a teaching module
- Three historical texts
- Essay assignments
- Experiences from a previous study
- A bit on the research methodology
- Some references
- Your reaction

Math program for Danish high school

- “demonstrate knowledge about the evolution of mathematics and its interaction with the historical, scientific, and cultural evolution”
- “demonstrate knowledge about applications of mathematics within selected areas, including applications in the treatment of a more complex problem”
- These educational goals may be accomplished through the so-called supplemental curriculum, which takes up 1/3 of the total teaching time

KOM: eight math competencies

1. Mathematical thinking competence
2. Problem solving competence
3. Modeling competence
4. Reasoning competence
5. Representation competence
6. Symbols and formalism competence
7. Communication competence
8. Tools and aids (ICT) competence

KOM: overview and judgment

- Three types of ‘overview and judgment’

1. Historically evolvement of mathematics, internally as well as in a societal context
2. The applications of mathematics in other practice and subjects areas
3. The nature of mathematics as a discipline [including philosophical aspects]

KOM: aspects of history of math

- How has mathematics developed through the ages?
- What were the internal and external forces and motives for development?
- What types of actors were involved in the development?
- In which social situations did it take place?
- What has the interplay with other fields been like? Etc.

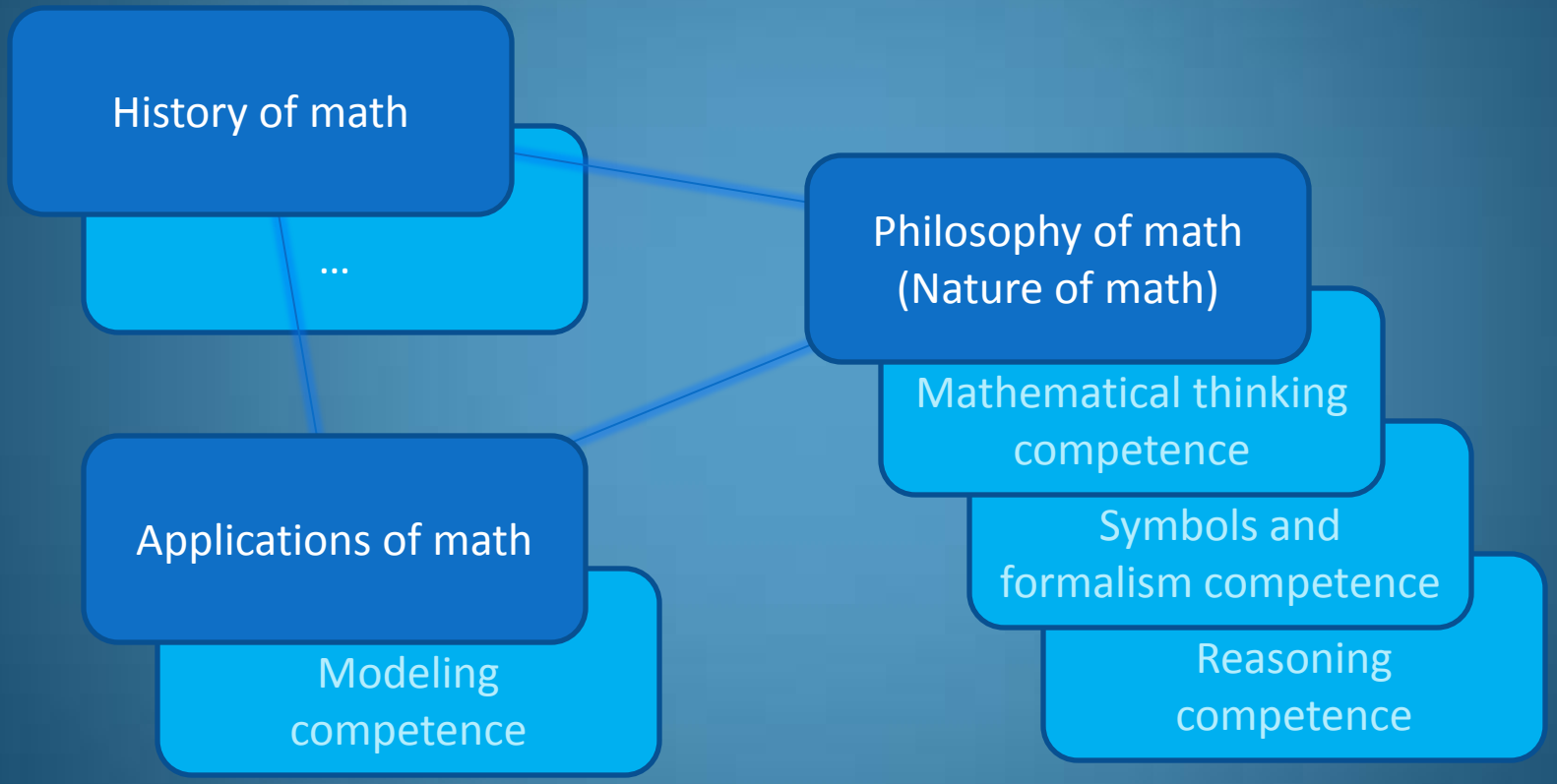
KOM: aspects of application of math

- Who, outside mathematics itself, actually uses it for anything?
- What for?
- Why?
- How?
- By what means?
- On what conditions?
- With what consequences?
- What is required to be able to use it?

KOM: aspects of philosophy of math

- What is characteristic of mathematical problem formulation, thought, and methods?
- What types of results are produced and what are they used for?
- What science-philosophical status does its concepts and results have?
- How is mathematics constructed?
- What is its connection to other disciplines?
- In what ways does it distinguish itself scientifically from other disciplines? Etc.

KOM: the interrelations



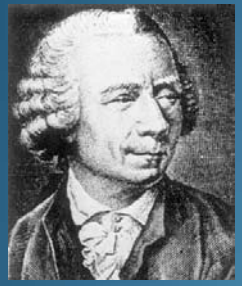
Question

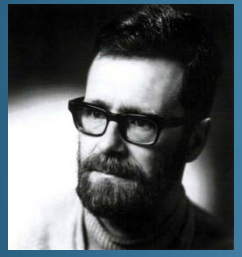
- How to design teaching modules that (more or less simultaneously) take into account all three types of ‘overview and judgment’, i.e. history, applications, and philosophy (HAPh) of mathematics, as well as the possible interrelations between the three?

Designing a teaching module

- An overall theme for the module that applies to all three types of 'overview and judgment'
- The choice: 'Mathematical problems'
- A decision of relying on readings of primary sources (e.g. easier to avoid Whigism)
- Striking a balance for the educational level (approx. 10^{th} grade, students of age 16-17 years)
- Finding a mathematical topic which presupposes only a minimum of related knowledge beforehand

The three selected historical texts





- Euler, 1736: *The solution of a problem relating to the geometry of position*
- Dijkstra, 1959: *A Note on Two Problems in Connexion with Graphs*
- Hilbert, 1900: *Mathematical Problems* (lecture at the 2nd international mathematician's congress in Paris)

Some elements of design

- A chapter for each historical source
- Following the design principles of the NMSU group (a guided reading of the sources with explanatory, historical, and modern math 'interruptions')
- Exercises along the way focusing on different mathematical competencies according to the type of 'overview and judgment' in play
- Students work in groups all of the time and the teacher circles around and provides help when needed
- The module is completed with the student groups doing a set of *essay assignments*

About the ‘essay assignments’

- ‘Essays’ are a way of having the students do reflections about *meta-perspective issues* of mathematics, its history, philosophy, and applications
- A key idea is to try and get the students’ discussions of these *meta-issues* *anchored* in their knowledge of the mathematical *in-issues* acquired through the teaching module
- Essay assignments are a way to bring about such anchoring of meta-issues in in-issues

Some results from a previous study

- A previous study focusing mainly on history, provides existence proof of it being possible to anchor students' meta-issue discussions in the related in-issues of a teaching module (and that such anchoring may occur on different levels)
- But in order for this to occur, a 'scene' must be set for the students
- The design of the modules and in particular the essays make up such a scene

Essays for this teaching module

- One on mathematical problems: mainly to be based on Hilbert's discussion of this
- One on mathematical proofs: why and how we proof things in mathematics, e.g. different proof types, counter examples, etc.
- One focusing on algorithms: their correctness, efficiency, usability in extra-mathematical situations, etc.

Essay on mathematical problems

- Identify the criteria which Hilbert poses for a good mathematical problem.
- How and to what degree do the problems treated by Euler and Dijkstra fulfill these? Provide examples and argumentation.
- Looking at the beginning of graph theory and its modern application for finding shortest paths and minimum spanning trees, how does this fit with Hilbert's description of the development of mathematics in general?

A bit on research design and method

- The same class of students will be given another teaching module also and the implementation of both modules will be monitored closely
- A combination of video filming the implementations, doing rounds of questionnaires and follow-up interviews with students and teacher, collecting written essay assignments and mathematical exercises
- The students of the class will be followed for a two-year period.
- Besides evaluating the design this is also done in order to try to provide answers to a set of research questions

Overall research questions of this project

What are the *connections between:*

a. Students' capabilities to discuss and reflect upon meta-issues of mathematics , related to the philosophy and history of math;
b. Their abilities to anchor these discussions and the related in-issues; and
c. The degree of consistency, justification, and exemplification in the students' beliefs about mathematics as a discipline.

And if any of the above develop and if the connections between them change in any way over a two-year period .

Some references

- Jankvist, 2010: *An Empirical Study of Using History as a 'Goal'*, **Educational studies in mathematics**, Online First.
- Jankvist, 2009: *A Categorization of the 'Whys' and 'Hows' of Using History in Mathematics Education*, **Educational studies in mathematics**, 71(3).
- Jankvist, 2009: *History of Modern Applied Mathematics in Mathematics Education*, **For the learning of mathematics**, 29(1).
- Jankvist, 2009: *On Empirical Research in the Field of Using History in Mathematics Education*, **ReLIME**, 12(1).
- Jankvist, 2008: *A Teaching Module on the History of Public-Key Cryptography and RSA,* **BSHM Bulletin**, 23(3).

Your comments and reactions

- What do you think about the design of the module?
- Do you see any pitfalls?
- Have you different ideas for topics for such essay assignments?

Thank you for your time

CERME WG12 on 'Hist in Math Ed'

Rezeszow, Poland, February 9-13, 2011

Group Leaders: Costas Tzanakis; Jan van Maanen; Snezana Lawrence; Uffe Th. Jankvist (Chair)

1. Theoretical, conceptual and/or methodological frameworks for including history in mathematics education;
2. Relationships between (frameworks for and empirical studies on) history in mathematics education and theories and frameworks in other parts of mathematics education;
3. The role of history of mathematics at primary, secondary, and tertiary level, both from the cognitive and affective points of view;
4. The role of history of mathematics in pre- and in-service teacher education, from cognitive, pedagogical, and/or affective points of view;
5. Possible parallelism between the historical development and the cognitive development of mathematical ideas;
6. Ways of integrating original sources in classrooms, and their educational effects, preferably with conclusions based on classroom experiments;
7. Surveys on the existing uses of history in curricula, textbooks, and/or classrooms in primary, secondary, and tertiary levels;
8. Design and/or assessment of teaching/learning materials on the history of mathematics;
9. Relevance of the history of mathematical practices in the research of mathematics education.