Fall 2010 Meeting of the
International Study Group on History and Pedagogy of Mathematics - Americas Section

<u>Saturday, October 23</u>

8:00-8:30 Registration

8:30-9:05 Patricia Baggett (New Mexico State University) and Andrzej Ehrenfeucht (University of Colorado), "Investigating concepts in mathematics textbooks over centuries using Google Books"

In spring 2010 we offered a university graduate mathematics course titled History and theories of mathematics education. For a final project each student gave a talk about a mathematical concept or topic, including its history and evolution as presented in actual textbooks used for instruction (K-12 and above). Students used electronic book sources that are now freely available online via Google Books. They thus had access to resources from authors "in their own words", many of which previously have been available only in rare book rooms of libraries and to expert historians--they were not restricted to secondary sources quoted by someone else. In this talk we report on some of the topics students selected (e.g., zero, multiplication algorithms, Euler's influence on American and British texts, the concept of derivative, reasons for teaching mathematics, introduction of manipulatives in early math instruction), the techniques they used, and some of their findings. We also discuss the characteristics of Google Books' online antiquarian textbooks and other historical mathematics education resources, and how they facilitated the students' research. And we report on the students' opinions of the course as given in anonymous evaluations, and plans for the future of the course.

9:10-9:45 Deepak Basyal (New Mexico State University), " Zero in the history of American arithmetic textbooks"

The realization of zero as a number has a long history. Initially, zero (cipher) was used solely as a placeholder; however, zero got its rightful place when it was abstractly defined in the New Math of the 1960s. Zero's journey from 'placeholder to cardinality of the empty set' and 'from sunya to zero' makes a really good story. In this presentation I will discuss the fact that the authors of arithmetic books throughout American history avoided using arithmetic problems involving zero. Why and how zero was treated throughout the 18th and 19th centuries, and how and when transition points in the meaning of zero appeared with the passage of time, are the main concerns of this presentation. It was a troublemaker when it first appeared; however, it is a vital member of the number system of arithmetic in modern mathematics.

9:50-10:25 Emily T.H. Redman (University of California, Berkeley), "Extracting Math from Science: A historical perspective on mathematics education reform efforts in 20th century U.S."

The history of mathematics education reform in the 20th century United States is often presented as part of the larger history of science education reform efforts that were accelerated in the wake of Sputnik. While this version of history does offer some value, it is imperative that the history of mathematics education be extracted from this larger narrative. Not only are concerted efforts in mathematics education reform more organized and more prolific in the years preceding the launch of Sputnik than those in science education, but also the post-Sputnik era shows mathematics education reform efforts to follow a unique trajectory. Indeed, while reform in mathematics and in the sciences both suffered failures and enjoyed successes, these need to be understood strictly along disciplinary lines. This paper aims to broadly outline the historically different reform efforts in mathematics education and begin to place them in a narrative separate from the traditional one that conflates math and science as a coherent whole. In doing so, past efforts can be better understood, as well as informing future educational reform measures.

10:25-10:40 break

10:40-11:15 David Dennis (San Bernadino, CA), "Mathematical Intentions: A Complete Ethnomathematical History of Our Current Secondary Mathematics Curriculum"

This talk will serve as an introduction to the new website " Mathematical Intentions," which will contain fifty half-hour lectures in mp3 form, many supporting documents in PDF form, and interactive dynamic geometry, spreadsheets, and algebra files done in Geogabra (a free software). All of this material will be free and easy to download. Most of it is already available at: www.quadrivium.info

The website takes apart every major idea in our current mandatory secondary mathematics curriculum (Algebra I – AP Calculus) and asks three historical questions.

1. What were the scientific, social, religious, political and economic intentions of the people who created this mathematics?
2. When and why were these mathematical ideas made into mandatory curriculum? and what parts of the original mathematics have we dropped from our curriculum?
3. Why do we still teach this curriculum? and how will it be transformed by new technology?

Thus for every mathematical idea taught in High School, these three different intentions will be described. Seemingly epic in scope, this task is easier than one might, at first, suspect since much of our mandatory mathematics curriculum comes from a rather narrow piece of social history. For example, the English Civil Wars of the 17th Century and their aftermath yield a huge amount of our current notions of school mathematics, and so the social history of the Puritan movement must be examined in detail, along with the Jesuit response. Once we understand their intentions we can ask much deeper questions about what pieces of their curriculum we still want to require of our students. Do we still believe that the natural world is "Gods other book?" Do we continue to accept the radical pragmatism and religious pluralism that led to Oliver Cromwell's "new model army?"

Intentions matter! especially when mandatory curriculum is tied to high stakes testing. This site is intended to get all of the relevant historical cards on the table as quickly and as efficiently as possible, specifically to aid in the reform of mathematics curriculum. Surprising answers are given to nagging questions that are glossed over in secondary curriculum, like "Where did fractional exponents come from? and what are they good for?" What was the role of tables, and how does that change with modern spreadsheets? Many historical examples are shown embedded in new transformational computer environments, e.g. animations of Descartes' curve drawing machines.

11:20-11:55 Shuhua An (CSU Long Beach) and Zhonghe Wu (National University, Costa Mesa), "Benefits and Challenges of Using History of Mathematics in K-8 Mathematics Classroom"

This presentation will address benefits and challenges of using history of mathematics in K-8 mathematics classroom. The presentation covers Issues in teaching mathematics, the role of history of mathematics in teaching and learning mathematics, and examples of history of mathematics in teaching and learning mathematics in K-8.

12:00-12:35 Barnabas Hughes (California State University, Northridge), "Three Useful Episodes in the History of Mathematics in Teaching Math"

The first two items, 1. *Foot Numbers from Fibonacci*, and 2. *The Cross Hatch Method of Dividing a Triangle in Several Parts,* appear in Fibonacci's *De practia geometrie.* The first is obviously a curiosity; the second is useful in a first year geometry course. The third offering, *Discovering Figurate Numbers*, fulfills a need for students to "do mathematics" in the classroom and at home. Ordinary word problems are too "recipe-ish" to be recognized as creative. *Discovering Figurate Numbers* provides many opportunities for students to discover mathematics by manipulating, tabulating and abstracting the results of their work. I shall demonstrate this activity.

12:40-1:15 Tom Apostol (Caltech), "Early History of Mathematics" (video)

This 30-minute video traces some of the important developments in the early history of mathematics, from Babylonian calendars on clay tablets produced 5000 years ago, to landmark events leading to the development of calculus in theseventeenth century.

It describes number symbols developed in different cultures; how numerology gave birth to number theory; the Pythagorean Theorem; the multicultural search for estimating the number π; how astronomy led to trigonometry; and efforts such as the creation of algebra and analytic geometry that accelerated the development of calculus.

It presents computer-animated demonstrations of the Pythagorean Theorem; the irrationality of the square root of two (a new geometric proof); the formula for the area of a circular disk; and the method of Archimedes for estimating π.

The video is enhanced by images of original documents, many from the Huntington Library, that help weave an historical perspective.

1:15- 2:15 Lunch

The remainder of the afternoon will be devoted to a visit to the Huntington Library.

<u>Sunday, October 24</u>

8:30-9:05 Colin McKinney, (Bradley University) "Adelphea: Archytas' Kindred Subjects and Echoes in Plato and Eutocius"

There is a fragment written by Archytas of Tarentum concerning the relationships of different mathematical subjects, which he called "kindred." Archytas is perhaps best known as a contemporary of Plato who first solved the famous duplication problem; indeed, we can see parallels between Archytas' fragment and a particular passage in Book VII of Plato's Republic. Centuries later, Eutocius of Ascalon quotes this fragment in his commentary on Apollonius when discussing compound ratios and the differences between logistic and arithmetic. In this talk, I will highlight these parallels in the context of ancient mathematics and ancient pedagogy.

9:10-9:45 M. A. (Ken) Clements and Nerida F. Ellerton (Illinois State University), "Rewriting the Early History of Mathematics Education in North America"

In this paper we will argue that published histories relating to the teaching and learning of mathematics in the north American colonies, and during the first 75 years of the united states of America, have over-emphasized the role of printed textbooks. We provide evidence from numerous contemporary sources that printed texts were less used and, from a mathematics teaching and learning perspective, were less important, than handwritten manuscripts prepared by learners. We have called these handwritten documents, "ciphering books."

After reviewing the European background to what we call the "ciphering tradition," we show how that tradition was translated into the north American colonies, and remained the dominant influence on mathematics teaching and learning until well into the nineteenth century.

9:50-10:25 Fred Rickey (United States Military Academy), "From the Mathematician's Study to the AP Classroom: 300 Years of Learning and Teaching Calculus."

The Bernoulli brothers, through dogged self-study in the eighteenth century, were the first to master the calculus of Leibniz. Countless others – famous names and nameless practitioners – followed their path from autodidact (in this arena) to teachers of calculus. But when did calculus start being taught in the classroom? Why did it happen? What textbooks were used? After a quick survey of the past, we concentrate our attention on the twentieth-century. In particular, we address the question of how calculus became a subject taught in the U. S. high school.

10:25-10:40 break

10:40-11:15 Maria Zack (Point Loma Nazarene University), "A Layman's Experience with Using Original Texts in the Classroom"

Like many mathematicians who teach courses in the history of mathematics, my degree is in theoretical mathematics and not the history or philosophy of mathematics. Over the last fifteen years I have been teaching courses on the history of mathematics as well as incorporating historical topics into a wide variety of mathematics courses in our curriculum.

In the last year I have experimented with using original historical texts (in translation if needed) in three classes: number theory, linear algebra and a course on mathematics, art and architecture. These experiments have included pilot testing some of the material created by the team at the New Mexico State University (David Pengelley, Jerry Lodder, et al) as well as creating activities of my own.

This talk discusses the successes, failures and lessons learned from a "non-expert" working with original texts in a number of settings.

11:20-11:55 Christine Latulippe (Cal Poly Pomona), "Advice from My History of Mathematics Course to Yours"

When teaching an undergraduate history of mathematics course, there are many considerations, especially when approximately half of the students enrolled are pre-service secondary math teachers. Whether it's modeling an interactive classroom, sharing views of math history that I hope students take into their own classrooms or broadening students' views of where math comes from, I utilize a variety of resources and activities in my teaching. I will share many of these that have worked well with my students. I will also include examples of student work which illustrate success in differentiating the post-secondary classroom.

12:00-12:35 Marty Bonsague (California State University, Fullerton) "A Hands-on Approach to the Great Theorems"

Math 380, History of Mathematics, fulfills the history and writing requirements for upper-division mathematics majors at Cal State Fullerton. Approximately two thirds of the students who take this course are pre-service high school or community college teachers of mathematics. Using the wonderful book *Great Theorems in Mathematics* by William Dunham as the primary text, the course aims to connect the content of cultural mathematics past with the content of school mathematics present. This presentation focuses on a "hands-on" approach to presenting some of these theorems, including the Theorem of Pythagorus (as presented in Euclid I.47) and Tartaglia/Cardano's Equation for the solution of the cubic $x^3 + mx = n$.

12:35-1:30 lunch

1:30-2:05 Jim Tattersall (Providence College), "How to Multiply Two Numbers Without a Calculator"

Multiplication is an ancient art and various techniques to multiply two numbers have been devised throughout history. Most are applicable to elementary or secondary school mathematics classes and provide alternate ways to multiply two numbers. We highlight the *kapta-samdi* or "junction of doors" method that has Hindus origin and is more efficient that the standard "zig-zag" method. The "junction of doors" method was introduced to English readers in 1726 by John Colson, Fifth Lucasian Professor of Mathematics at Cambridge. Its justification was given by Cauchy in 1821. Before focusing on the 'junction of doors" method we illustrated several other ways to multiply two numbers.

2:10-2:45 Stacy Langton (University of San Diego), "Tschirnhaus's Method of Elimination"

In 1683, Ehrenfried Walther von Tschirnhaus published a paper "Methodus auferendi omnes terminos intermedios ex data aequatione" ("A method for removing all the intermediate terms from a given equation") in the Leipzig Acta Eruditorum. Given an equation f(x) = 0, where f is a polynomial of degree n, Tschirnhaus's proposal is to make a change of variable y = g(x), where g is a polynomial of degree n - 1. By eliminating x between the two equations, Tschirnhaus gets an equation h(y) = 0, where h has degree n, and by choosing the polynomial g appropriately, the equation h(y) = 0 can be made to be a binomial equation, which can be solved by taking the nth root. (We say nowadays that the equation h(y) = 0 is obtained by making a

Tschirnhaus transformation.)

2:45-3:00 break

3:00-3:35 Harriet Lord (Cal Poly Pomona), "Surface Area, Arc Length, and Functions of Bounded Variation"

As a young student, Henri Lebesgue was first intrigued by arc length when he learned of a simple example that showed that if a sequence of polygonal curves converges uniformly to a given curve, the corresponding sequence of lengths of those polygonal curves does not necessarily converge to the length of that curve. His interest was rekindled by the Schwarz's Paradox, which showed that if a sequence of inscribed polyhedra converges uniformly to a surface, the areas of those inscribed polyhedra does not necessarily converge to the area of that surface. We investigate the work of Lebesgue on Arc Length and Surface area, as well as that of Peano, who claimed priority in the publication of "Schwarz's Paradox". We then trace the definition of arc length back to Jordan and investigate his work on rectifiable curves. It was in his study of rectifiable curves that Jordan used his concept of Bounded Variation.

3:40-4:15 James T. Smith (San Francisco State University), "Tarski, Schools, and Geometry"

Alfred Tarski (1901-1983) perfected our framework for research in mathematical logic. For decades, as a Berkeley professor, he was the preeminent figure in that field. This talk continues Prof. Roman Sznajder's March 2010 HPM talk on the emergence of the Polish School of Mathematics. I'll sketch Tarski's upbringing and schooling in Poland, emphasizing the chaotic social context. Then I'll depict Tarski's dual roles in geometry, as mathematical researcher and Warsaw schoolteacher. I'll note where needed historical work is underway.