The West Coast Meeting of the Americas Section

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Abstracts of Talks

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TITLE: A Story Problem about a Privateer in Nineteenth Century Calculus Books

ABSTRACT: A privateer was a private ship employed by a government to attack and capture enemy vessels. It was paid by looting the vessels it captured. England used privateers against Spain, the United States against England, and the South against the North during the Civil War, even after the Declaration of Paris banned privateering in 1856.

A calculus problem dealing with equal illumination and other mathematically related questions of physics has been present in American calculus books since (at least) 1826. In 1854 it was recast as a story problem about a privateer's safe passage. In this format it was present in books of six different authors (Smyth, Olney, Bowser, Taylor, Hall, and Mellor) until 1902. Later it returned to the original (physics) version.

[2] James T. Smith (with Andrew Sznajder & Joanna McFarland)
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Professor Emeritus of Mathematics
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TITLE: Alfred Tarski: Problems for Students and Teachers

ABSTRACT: Alfred Tarski (1901-1983) perfected our framework for mathematical logic. As a Berkeley professor, he became its preeminent figure. During 1924-1939 he was a full-time schoolteacher in Warsaw, as well as part-time University lecturer. During 1930-1932 he contributed three papers and a set of fourteen problems to the Polish journal *Parametr*. This impressive but short-lived publication targeted teachers of mathematics in the schools, and their best students. I'll describe the journal and some representative problems of Tarski and address the questions, what was their intent, and were they effective?

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[3] Nicole M. Enzinger
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TITLE: Subtraction Algorithms from 1700 to 1900

ABSTRACT: From the 1700s to the 1900s at least three subtraction algorithms were being used: the equal additions algorithm, the decomposition algorithm, and the complementary method. Other algorithms such as the Austrian algorithm may also have been used. These four algorithms will be described in this paper. Terminology issues such as "substraction" versus "subtraction," "minuend" and "subtrahend," were also present, although not uniformly, throughout the 1700s and 1900s. In the mid-1800s methods used by teachers in common schools had a direct influence that shaped student learning, affecting not only the language and terminology used, but also the choice of algorithms. Despite textbook authors' increasing support of the decomposition algorithm between 1700 and 1900, the equal additions algorithm and the complementary method for subtraction remained the most used algorithms from the 1700s to the early 1900s. This paper describes the changes which occurred and attempts to explain why they occurred. Finally, this paper presents interesting data with respect to "proofs" and "markings" that were found in the books of this period.

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[4] Robert Ely
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TITLE: Dimensional Collapse in Calculus

ABSTRACT: Research indicates that calculus students often imagine objects losing a dimension entirely when a limit is taken, and that this image serves as an obstacle to their understanding of the fundamental theorem of calculus. I argue that similar imagery in the form of "indivisibles", used by mathematicians such as Cavalieri, Galileo, Torricelli in the mid-1600s, was similarly unsupportive of the development of the fundamental theorem. I discuss how reasoning with infinitesimals instead of indivisibles promoted understanding of the fundamental theorem, and how this imagery can likewise support students to overcome the obstacle of dimensional collapse. This account suggests that the more modern approach with limits can also work to surmount the dimensional collapse obstacle, but only if focus is carefully maintained upon quantitative reasoning.

[5] Jessica Pierson Bishop (with Lisa Lamb, Randolph Philipp, Ian Whitacre, Bonnie Schappelle, & Mindy Lewis)
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San Diego State University, Center for Research in Mathematics and Science Education

TITLE: Historical-Mathematical Parallels in Children's Reasoning About Integers

ABSTRACT: In this presentation we document instances of how children's thinking about integers has paralleled that of mathematicians of the past, highlight ways in which children have displayed expert-like ways of reasoning about integers, and provide explanations for why some children are able to reason about integers in reasonably sophisticated ways. By considering the ways that children reason about integers and the difficulties they face in extending their numeric domains from nonnegative to negative integers from a historical perspective, we seek to problematize the rich and nuanced understandings of integers we in the mathematics community hold.

[6] Ian Whitacre (with Jessica Pierson Bishop, Lisa Lamb, Randolph Philipp, Bonnie Schappelle, & Mindy Lewis)

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San Diego State University & Center for Research in Mathematics and Science Education

TITLE: Integers: History, Textbook Approaches, and Children's Productive Mathematical Intuitions

ABSTRACT: We juxtapose the history of integers with current textbook approaches to integer instruction. We also present findings concerning children's reasoning about negative numbers prior to instruction. We see alignment between children's intuitions and the avenues that afforded progress for mathematicians, while the textbook approaches tend to run counter to lessons learned from history.

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[7] Gizem Karaali
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TITLE: Purpose and Humanism in Mathematics Education Research 1968 – 1996

ABSTRACT: One of the most influential journals in mathematics education research opened with an editorial titled "Why to teach mathematics so as to be useful" [Hans Freudenthal, *Educational Studies in Mathematics*, Volume 1 (1968), pp. 1-2, 3-8]. Thus began an extended discussion on the purpose(s) of mathematics education that continued across many years and volumes, though mainly appearing as one undercurrent or hidden assumption among many. In our daily lives as mathematics educators, we often confront this same question directly: Why should I learn mathematics? Though this version frequently comes coated in subtle hostility toward the subject and may sometimes be cast aside as such, the underlying question is still worthy of our scrutiny and understanding: Why do we teach mathematics?

This paper focuses on this question and, via a thorough analysis of the first 30 volumes of the above-mentioned journal, attempts to document how attitudes toward purpose evolved amidst mathematics education researchers of the period. In particular, we note in our study the emergence and development of the humanist and social constructivist paradigms on the one hand, and the interlocked themes of discovery, inquiry and active learning in the classroom on the other, and analyze how their proponents engaged with the question of purpose. More specifically we look at when and how the student/learner comes under the lens of the mathematics education researcher as a human being with individual emotions, intentions, and values, and whether and how this merges

with the professional stance of the researcher on the question of purpose. The time spanned in these thirty volumes takes us from 1968 to 1996, approximately three decades where many different trends have come and gone in the field. However the thread we follow is relevant up to this day, and is interwoven into the fabric of our existence.

[8] Kathleen M. Clark
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TITLE: Reflections on Historical Problems and Problem Solving

ABSTRACT: In an effort to capture ways in which prospective secondary mathematics teachers (PSMTs) engage in solving problems found in historical sources during a history of mathematics course, I implemented a 'problem solving portfolio' assignment. Each PSMT selected ten problems that they previously solved in the course, presented their solution, and provided a reflection on how their work on each problem contributed to the ways that they considered the underlying mathematics addressed in the problem. In this talk I will share several of the problems that were most often selected by the PSMTs and highlight the common themes raised by them in their reflections.

[9] Janet L. Beery
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TITLE: Thomas Harriot's Cannonball Curves and Other Animated History on the Web

ABSTRACT: Can modern technology help students understand mathematical thinking from the past? We show how animated graphics, in particular, can help students visualize models of projectile motion of Thomas Harriot (1560?-1621), measurement of the volume of a wine barrel by Johannes Kepler (1571-1630), multiplication using Napier's bones by John Napier (1550-1617), and even a sum of squares formula of Archimedes (3rd century BCE). These animated illustrations of the history of mathematics come from the National Curve Bank, MatematicasVisuales, and MAA Convergence.

[10] Kurt Kreith
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Department of Mathematics
University of California at Davis

TITLE: A Copernican Metaphor: Using History and Technology to Teach about Global Change

ABSTRACT: The California State Summer School for Mathematics and Science, better known as Cosmos, offers programs for talented high school students at four UC campuses. In offering a course at the Davis Cosmos site (http://cosmos.ucdavis.edu/cluster6.htm) I have, with varying degrees of success, dealt with the role that mathematics and computer technology might play in addressing issues of global environmental change.

My 2011 course, entitled "The Mathematics of Global Change", takes advantage of a fortuitous event. On October 9, 2010 Terence Tao gave an AMS Einstein Lecture entitled "The Cosmic Distance Ladder" (http://www.ams.org/meetings/lectures/einstein-2010). Here Tao describes the use of high school mathematics (geometry, proportional reasoning) in developing a sense of scale for the earth and solar system, and the ways in which such celestial numeracy led to an understanding of celestial change (notably Kepler's laws and Newton's derivation of these laws from an inverse square law of gravitation).

Aside from its inherent interest, this account suggests a way of approaching issues of global change. Starting with Eratosthenes' measurement of the size of the earth, one can cultivate a form of global numeracy that leads rather naturally to questions about global change. In developing mathematical tools intended to address such questions, one can also seek analogies between the dramatic events that accompanied the Copernican revolution and the issues we are likely to face in coming to terms with "our global trajectory".

This talk calls for a description of such a course and an effort to assess its strengths and weaknesses.

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[11] Stacy Langton
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TITLE: Bernoulli's Method for Polynomial Equations

ABSTRACT: A topic treated in some textbooks on Numerical Analysis is "Bernoulli's method" for the numerical solution of polynomial equations. Although it does not converge as rapidly as "Newton's method", Bernoulli's method has the advantage that it does not require

a starting guess for the solution. Daniel Bernoulli discovered the method around 1723 while he was in Venice, studying medicine. After he became Professor of Mathematics at the newly founded St. Petersburg Academy of Sciences in 1725, Daniel published the method in the Academy's journal. Later, Leonhard Euler gave an exposition of the method in the first volume of his great textbook *Introduction to Analysis of the Infinites*. In this talk, I will show how the method works, and I will explain what led Daniel Bernoulli to the discovery of the method. (This is based on joint work with Dan Kalman of American University, Washington, DC.)

[12] Kishore Marathe
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City University of New York, Brooklyn College

TITLE: Physical Mathematics: Old and New

ABSTRACT: From ancient times to the 19th century mathematics and physics were studied under the area called natural philosophy. Major contributions to both the fields were made by the same scientists. This picture changed dramatically in the 20^{th} century due to rapid progress and divergence of methods of inquiry in both the fields. This situation has now started to reverse itself in the last 30 years. We will discuss these new developments and review old results where physical theories influenced the development and pedagogy in mathematics.