The Varieties of Relationships to Mathematics of the Past

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*Introduction*

I would like to begin this paper autobiographically. When I started to work on the history of mathematics, I do not think I gave too much thought as to the nature of the subject. I liked mathematics and I liked history; I also liked biography and I liked the Greeks, so I started studying the mathematician Apollonius of Perga. I worked with the historian of mathematics Sabetai Unguru. Because of his own work and his own example I could hardly avoid thinking about what it meant to do the history of mathematics and what it meant to be an historian of mathematics. By the time I finished my Ph.D., I could make some distinctions: I could divide historians of mathematics into the mathematician type, such as Zeuthen or van der Waerden, the historian type, like Sabetai, and, perhaps, a postmodern type.

Even this distinction is useful, but it is also quite crude, especially since the last possibility is hardly to be taken seriously! I have realized just how crude it is recently in the context of some work I have done on Edmond Halley’s reconstruction of Apollonius’ last book of the *Conics*. In this connection, I have had to ask what it meant for Halley in 1710, when powerful new mathematical tools were being developed, tools which Halley himself had a part in making and certainly mastered, what it meant *then* for Halley to turn his attention to a work of *ancient* mathematics. Although his text was from the past, it is not clear Halley’s endeavor could be characterized as purely historical in the modern sense; but nor was it completely unhistorical, like the much earlier attempt to reconstruct *Conics, Book VIII* by Ibn al-Haytham. The more one considers the question the more it becomes evident not so much that Halley’s way of treating ancient Greek mathematical works was *sui generis*—for it was not—but simply that there are many different kinds of relationships to mathematics of the past.

The main goal of this paper is to chart out these many relationships. Of course I cannot discuss in depth all the ways one can stand with respect to the mathematics of the past and still keep this paper a reasonable length. But what I would like to do is to list some of the more important of them, almost as an expanded table of contents. I hope that by doing so I will at least make the value of such a typology of relationships plausible. In particular, I would like to suggest how this kind of typology can form a natural bridge between history and historiography, clarifying on the one hand ways in which one can treat mathematics of the past, while, on the other, clarifying how mathematicians of the past viewed their own sources and their own position with respect to them. As for the latter, the historical side of the equation, I should emphasize I am not looking at the actual relationship between mathematicians and their sources—a standard task for historians of mathematics. What I am interested in is how those who have concerned themselves with mathematics one way or the other viewed the past: I grant the line

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1 The title was inspired by William James’ famous work, *The Varieties of Religious Experience*. 
between how texts of the past were used by such people and how they stood with respect to the past can be quite fine, but I believe there is indeed a line.

So this catalogue is one goal. But I have another goal as well, one that has to do with education. For as one interested in the role of history of mathematics in mathematics education, I have been long preoccupied with the questions, what does the mathematics of the past mean to students and what can it mean to them? Should our relationship to the past, as educators, be aligned with one of those I shall outline in what follows, and therefore have as much variety as they represent, or is it a unique relationship different from all the others? In other words, is there really a distinct educational relationship to our mathematical past? And, if so, does it sit beside the others as an equals, or perhaps, does it need to be represented on a different axis, a different dimension of the relationships, to use a somewhat hackneyed expression? I will not answer these questions, but I will describe one possible such relationship—not necessarily one that approve of, rather, one that can focus the questions about an educational type. It may be that our job as educators, in fact, is to assure that these questions remain open so that our students can develop their own relationship to the past.

*Historical and non-historical postures towards mathematics of the past*

Looking over the totality of relationships or postures towards mathematics of the past, the first and most basic division one should make is between those postures that are historical and those that are non-historical. It is, actually, not always an easy distinction to make, especially when it comes to mathematics. This is partly because of the seductive tendency to see mathematics, at bottom, as ahistorical, that knowing some piece of mathematics occurred in the past has little to do with its meaning. And, on the other end, it is partly because of the seemingly-common-sense view that speaking about the past is *ipso facto* engaging in history.

In this connection, I have found Michael Oakeshott’s view of the historical mode of experience useful. Oakeshott makes the point that although “History is certainly a form of experience in which what is experienced is, in some sense, past...history is not the only past, and a clear view of the character of the past in history involves the distinction of this past from that in other forms of experience” (Oakeshott, 1933/1966, p.102). The past that, in Oakeshott’s view, stands in clearest contrast to the historical past is what he calls the “practical past” (p.103). This past, like the historical past, is an experience in the present, unavoidably; however, it is a past that is *for the sake* of the present. One might say that the subject of the “practical past” is the present even while it refers to the past. The subject of the historical past, however, is the past as distinguished from the present, the past in its own particularity (p.106). The “practical past” sounds very much like Butterfield’s “Whiggism,” but, unlike Butterfield who rejects the Whig interpretation as illegitimate, *tout court*, Oakeshott is willing to see the “practical past” as simply different from the “historical past.” It is this aspect of Oakeshott’s view that makes it particularly appropriate for thinking about relationships towards mathematics of the past, for it allows us to see the historical and the non-historical as two reference points for locating these relationships without judging them as necessarily legitimate or illegitimate.
I ought to mention in passing another similar distinction proposed by Ivor Grattan-Guinness (2004a,b) specifically regarding mathematics, namely, that between “history” and “heritage.” These are distinguished by their guiding questions: for history it is, “What happened?” or “Why did N happen?”; for heritage the question is “How did we get here?” where the answer, Grattan-Guinness playfully points out, is more often than not via, “The royal road to me.” Like Oakeshott, Grattan-Guinness wants to make the point that referring to heritage as if it were history or history as if it were heritage is to fall into the trap of ignoratio elenchi, as Oakeshott would have put it. There is no doubt Grattan-Guinness’s history-heritage dichotomy can be a useful tool for analyzing how mathematics of the past is treated; however, for my own purposes, it is not broad enough and, more importantly, it does not bring out explicitly enough how these different approaches to the past are in fact different views of the past itself and places one in a different relation to the past. For this reason, in my book on Apollonius with Sabetai Unguru, we spoke about going through a historical door or a mathematical door (Fried and Unguru, 2001, pp.404ff). Each door leads into a very different world: “The mathematical and the historical approaches are antagonistic. Whoever breaks and enters typically returns from his escapades with other spoils than the peaceful and courteous caller” (p.406).

Mathematicians, Mathematician-Historians, Historians of Mathematics

But in dividing the historical and non-historical relationship with the past, we ought to take care not to make the discussion only one of good and bad history. One can enter the world of past mathematics with no real intention to interpret the past, that is, with no real intention of doing something like history; rather, returning to Oakeshott’s “practical-past,” a mathematician can see mathematics of the past as a resource, referring to it as one might refer to a past issue of a contemporary journal. For such a mathematician, the past is past, but only in name, something incidental. On the other hand, those who come through the mathematics door, yet who see themselves investigating history, in some way may live in the practical past, but they have a different relation to the past than those who use the past directly for mathematical work.

With that, we can see three initial categories with respect to the basic division between non-historical and historical postures towards mathematics of the past. Towards the non-historical pole, we have what I shall call simply “mathematicians,” for they are just that, people who see themselves doing mathematics, not history; towards the historical pole, we have “historians of mathematics,” for these see themselves doing history and their mode of experiencing the past is historical in the Oakeshottian sense, that is, they relate to the past as something utterly apart from the present, the past as a problem; ranging the middle we have “mathematician historians,” for these are generally mathematicians who see themselves engaged in an historical enterprise and yet to a greater or lesser degree (and there are many such degrees!) see a continuity between the mathematics of the past and their own mathematical work.

Mathematicians

Before discussing these categories and their subtypes, I should set out some caveats. First, what I call “mathematician” refers only to a type of relationship to the past: to be
included in this type does not require one to be a mathematician in the usual sense, nor
does it mean that if one is a mathematician by training then one is necessarily a
“mathematician” by type. Second, these types, in general, should not be viewed in an
absolute way: they are a little like Weber’s ideal types—merely means of analyzing
relationships. Finally, although the presentation will follow a more or less chronological
pattern, “mathematicians,” “mathematician-historians,” and “historians of mathematics”
are not necessarily themselves historical categories: one can be a “mathematician
historian” today as well as in the 18th century. I will though progress steadily from the
non-historical to the historical pole. With that out of the way, let me begin with
“mathematicians,” of which I want to distinguish three subtypes.

The first includes what I call “mathematical colleagues.” In this case, figures in the past
are viewed as if they were contemporaries working in the same field and working,
fundamentally, in the same way. Thus one feels fully justified referring to
mathematicians of the past as Littlewood famously said of the Greek mathematicians,
namely, as “Fellows of another college” (quoted in Hardy, 1992, p.81); mathematician’s
of the past, like one’s colleagues, are useful for gaining insights into one’s present
mathematical research. This is the kind of relationship one sees in Apollonius’ references
to Euclid and Pappus’ references to Apollonius and Euclid. The example of Apollonius
is less problematic than that of Pappus because of the relatively small amount of time
separating Apollonius and Euclid, probably somewhat more than a half a century.
However, whatever chronology one chooses to use, it is safe to assume that they were not
contemporaries. Yet, when Apollonius refers to Euclid’s attempts to solve the locus
problem, Apollonius does not see himself having the benefit of modern methods but only
of more powerful propositions, which he himself discovered. In other words, Apollonius
views Euclid working on the same problems and within the same basic framework as he
is: his criticism of Euclid is a little like the criticism of colleagues in the literature review
of a research paper. Apollonius’ “historical” comments then, are really just a way of
presenting his own accomplishments and originality within the same context as Euclid.
Indeed, Jaap Mansfield (1998), discussing the nature of Greek mathematical
introductions, makes the point several times that “historical” remarks are a routine part of
setting out the skopos or theme of the work. In this light too, Pappus’s comments at the
start of Book VII of the Collection where he castigates Apollonius about not giving
Euclid enough credit in connection to the locus problem should really be viewed as a way
of clarifying the present theme of the work, even though it appears to be a matter of
historical judgment. After all, Pappus’s purposes were not historical: he was expounding
a body of knowledge meant for “those who want to acquire a power in geometry that is
capable of solving problems set to them” (Jones’ translation: Jones, 1986, p.82) and
Euclid and Apollonius are the central figures for this non-historical project.

The next type, “mathematical treasure hunters,” is very similar to the first in that such see
themselves working within the same framework and on the same type of problems as the
mathematicians of the past to which they refer. The difference stems from the great span
of time separating them from the latter. The effect of this span of time is that the
mathematics of the past is for them to some extent lost and needs to be found or
recovered. This type actually can be divided further, for there are those that see
themselves as somehow inferior to the mathematicians of the past and those who see
themselves as their equals. Both, however, seek to find lost treasures. But the word
“treasure” needs to be qualified. A mathematical treasure is a thing to be understood; effort is required from the treasure hunter to piece together the mathematical text. Thus, mathematical treasure hunters see themselves continuing or completing the work of the ancients, whether or not this is truly the case, as Sabra famously discussed in the case of Islamic mathematical treasure hunters (Sabra, 1987).

In fact, it is certain streams of Islamic mathematics I have particularly in mind here, and the image of “treasure hunters” actually comes from thinking about the Banū Mūsā searching the world for ancient mathematical manuscripts. But a more nuanced example is Ibn al-Haytham, at least with respect to his reconstruction of Apollonius’s *Conics*, Book VIII, which I mentioned earlier. Al-Haytham called his work a *Completion of the Conics* because, as he puts it in his introduction, “When we studied this work [the *Conics*], investigated the notions in it, and went through the seven books many times, we found that it lacked notions, which this work *should not leave untreated* [my emphasis]” (Translation from Hogendijk (1985), p.134). The argument for the rationale of the reconstruction was, in effect, that these missing notions were worthy of Apollonius, so that not appearing in the extant books of the *Conics*, they had to have appeared in the lost book. What is important though is that Ibn al-Haytham could consider himself able to judge what was worthy of Apollonius because he saw himself as a fellow mathematician and one whose own thoughts about conics were consistent with any of those Apollonius might have thought. As Hogendijk (1985) says, “Ibn al-Haytham’s supposition [regarding the “notions” he judged to have been necessarily included in Book VIII] is that Apollonius gave a complete treatment of certain classes of related problems. It seems to me that he based this supposition not on evidence in the *Conics*, but only on his implicit assumption that Apollonius’s interests were identical to his own” (p.69). Like the “mathematician” type, then, Ibn al-Haytham views Apollonius’s and his own mathematical thought as coterminous intellectually. So, while “mathematical treasure hunters” may be looking back at a work of the past, they use it with an eye to producing a work on the frontier of new knowledge; the antiquity of original works is an almost incidental matter, except, perhaps, that one’s present work *might* have been done previously before being lost; one might as well refer to the original works as the precious manuscripts of a brilliant colleague who has died and whose work was lost in a fire.

The next type, which I call “mathematical conquerors,” is in the transitional area between “mathematicians” and “mathematician-historians.” Unlike the “mathematical treasure hunters” who tend to feel either inferior or equal to the mathematicians of the past, “mathematical conquerors” see themselves as equal or superior to those ancient mathematicians. They see the mathematics of the past as an opportunity to highlight their own originality and power. But more than an isolated theorem or set of theorems, they see themselves possessing general methods and approaches that allow them to open doors the ancients left shut: their power is conceived as power *over* the mathematicians of the past.

Descartes, as one might guess, is a perfect example of this type. It is Descartes the “mathematical conqueror” speaking in the *Regulae ad Directionem Ingenii* when he says that there were “traces” (*vestigia*) of a general method in Pappus and Diophantos that they hid by “a certain pernicious cunning” (*perniciosa quadam astutitia*) (Reg. IV.5). He refers the ancient world as “that unsophisticated and innocent ancient time” (*rudi ista et
pura antiquitate); it is Descartes himself who is cognizant of the true depth of the method, which he identifies with algebra: he has no need to hide it jealously. This sense of possessing a key that no locked door can resist is also behind the flurry of reconstructions at the beginning of the 17th century (I might add that one can discern these types through the variety of reconstructive efforts in the history of mathematics and their motivations). Examples include François Viète’s reconstruction of Apollonius’s On Tangencies, his Apollonius Gallus of 1600 and Pierre de Fermat's reconstruction of Apollonius’s Plane Loci, which he worked on from about 1628 to 1636. Regarding the latter, Michael Mahoney (1994) summed up the situation as follows:

Fermat was no antiquarian interested in a faithful reproduction of Apollonius’ original work; he was a working mathematician seeking to ferret out the analytic techniques he felt Apollonius had hidden. The Plane Loci was to serve as a means to an end rather than an end in itself (p.96).

But again, I want to emphasize that in “...ferret[ing] out the analytic techniques he felt Apollonius had hidden...” Fermat and others of his type were actually showing their own possession of powerful mathematical methods. In this sense, these reconstructions at the start of the 17th century were in fact pressing the development of mathematics forward.

The “mathematical conquerors” are, as I have said, a transitional category, not of course in time but in type. They are located in the general category of “mathematicians” because they see themselves engaged in an enterprise meant to further the development of the methods and ideas that they themselves are exploring in their own mathematical work. Undeniably, though, they also have a sense of the past and feel they are explaining the past. On the other hand, their sense of the past has the unambiguous character of a “practical past,” again to use Oakeshott’s term.

Mathematician-Historians

“Mathematician-historians,” the next basic category, are perhaps the hardest to describe. They are themselves well-trained mathematicians, but they stand apart from “mathematicians” because they have a fuller sense of the mathematics of the past as past. At the same time, past mathematics is understood by them as continuous with present mathematics; however, the mathematics of the past may be looked upon dispassionately because progress in contemporary mathematics does not require them to look back to the past. “Mathematician-historians,” accordingly, do not see their mathematical work as utterly dependent on their understanding of past mathematical work; on the other hand, they do see that mathematics as a discipline and, more pointedly, their own identity as mathematicians are elucidated by such understanding. For this reason, these do have a foot planted in the direction of history and deserve the word “historian” in their description—some to a greater degree than others. As with “mathematicians,” there are three subtypes in this case.

The first subtype, furthest from the historical pole, is what I call, “moderators.” Like all other types in this category, “moderators” are completely mindful of the advantages of the mathematics of their own day over older mathematics; they no longer need to prove the potency of algebraic methods, for example. However, they have studied older, usually classical works of mathematics, know them well, and respect their authors. They are not
interested, accordingly, in conquering the mathematics of the past—for they no longer need to—but to moderate a conversation, as it were, between ancients and moderns.

It is in this group I place Edmond Halley, and I take him as my main example, though he is hardly the only figure who could serve the purpose: probably Barrow could also be considered “moderator,” maybe even Newton. The time was right for such “moderators,” for the battle between the “ancients and moderns,” as immortalized in Swift’s famous satire, “The Battle of the Books,” was raging, and where there are battles there will eventually be moderators. Halley’s official appointment as Savilian Professor of Geometry at Oxford put him in the position of being a “moderator,” since the professorship, besides the usual duties of a mathematics professor, required Halley to lecture on Euclid, Archimedes, and Apollonius. But Halley took up these responsibilities, it seems, willingly and not as a chore. Throughout his life he took great delight in classical works and in history, especially when he could relate his scientific knowledge to them (see Fried (in press) and Chapman (1994)). His identity as a moderator can be seen very clearly in the preface to his earlier translation of Apollonius’s Cutting-Off of a Ratio and reconstruction of Cutting-Off of an Area (1706), which he opened by extolling the modern achievement of the “Algebra of Species,” the “Arithmetic of Infinitesimals,” and the “Fluxions,” referring to the works of Viète, Wallis, and Newton, but then continued by urging that this should not in any way lessen the glory of the ancients who brought geometry to perfection (...qui Geometriam ad eam provexere perfectionem). This role as “moderator” can be seen also in the actual reconstructions themselves. In his reconstruction of Conics VIII, for example, he adopts Apollonius’s voice as best as he can in the statements of the problems and their proofs; however, having completed the problem as he supposes Apollonius might have done, he adds his own solution. Thus Apollonius’s problem 7 in Halley’s reconstruction Conics, Book VIII, reads:

Given the axis and the latus rectum of the axis of a hyperbola, and given the ratio of conjugate diameters of the section, find the conjugate diameters both in magnitude and in position [my emphasis].

But then, having presented his proposal for Apollonius’s solution, Halley adds:

Since the difference between the squares on the conjugate diameters is always equal to the difference between the squares on the axes, though, we can give this solution to the problem in a fairly expedient way (modo satis expedito), but without the position of the diameters [my emphasis].

As in this case, Halley’s solution is almost always one that emphasizes only the magnitudes of lines and not their position, that is, the aspect of the problem that would lend itself to the kind of analytic emphasis of his own modern mathematical world. There is no hint that these alternatives are meant to show how a modern like himself can outdo Apollonius: it is a kind of dialogue.

The next two subtypes are closely related. They are the “privileged observers” and “mathematical critics.” What distinguishes both of these subtypes is that they not only deem their modern mathematical knowledge to be superior to the mathematics of the past but also believe it provides them special power in interpreting the past. In this way, they have kinship with “mathematical conquerors”; however, their end is not to prove the superiority of their mathematical ideas but to take advantage of that superiority to piece
together the past. For the “privileged observers” the latter is the principal goal. For the
delicate critics,” though, understanding is not enough: one must also show how
mathematics of the past has made a positive contribution to present day mathematical
truth and how it has not, where it was right and where it was wrong, by modern
standards.

I take Hieronymus Georg Zeuthen as a good representative of the subtype “privileged
observers,” although I could have as easily chosen André Weil or Bartel Leendert van der
Waerden. About Zeuthen, Lützen and Purkert (1994) say that he, “…always stressed that
he made his contributions to this field [history of mathematics] not as a historian but as a
professional mathematician” (p. 14). In his most well-known historical work, Die Lehre
von den Kegelschnitten im Altertum, accordingly Zeuthen makes it clear that it is indeed
his modern methods, his being a modern mathematician, that provides him with a
privileged standpoint for understanding Apollonius’s Conics, the main subject of his
1886 work. Thus he writes that a proper view of the work can be achieved only “…by
employing modern means of representation,” which, unfortunately, “…the exclusive
preoccupation of the ancients with logical completeness conceals” (Zeuthen, 1886, p.
xii). But Zeuthen, by saying this, was not trying to make a case for his modern means of
representation, that is, he was not using history as a vehicle for advancing an area of
mathematics as one would expect from a “mathematical conqueror”: there is no reason to
believe that Zeuthen ever thought he was doing anything but history.

An example of a “mathematical critic” is Clifford Truesdell. As an historian, Truesdell
brought to his work immense and exacting mathematical and scientific insight, but his
approach was generally to show where Euler, Lagrange, the Bernoullis, and the others he
studied got it right and where they got it wrong—and right and wrong, in his view, were
to be taken as absolutes: the same today as yesterday. This sounds very much like the
subtype I called a “mathematical colleague”; I might have categorized Truesdell this way
if he did not present what he was doing as history and distinguishable from his purely
scientific work. Yet Truesdell’s case does show how unclear the picture can be. For
example, about history Truesdell has written:

One of the main functions [the history of mathematical science] should fulfill is to
help scientists understand some aspects of specific areas of mathematics about which
they still don’t fully know. What’s more important, it helps them too. By satisfying
their natural curiosity, typically present in everybody towards his or her own
forefathers, it helps them indeed to get acquainted with their ancestors in spirit. As a
consequence, they become able to put their efforts into perspective and, in the end,
also able to give those efforts a more complete meaning” (in Giusti, 2003, p.21)

Nothing could be a clearer picture of Oakeshott’s “practical past” (or better Grattan-
Guinness’s “heritage”) than that, and it is a picture that comes very close to the
“mathematician” category of relationships with mathematics of the past. The case of
Truesdell, thus, is also a good opportunity to remind the reader that the categories and
subtypes I have described must be taken only as “ideal types,” and not as complete
descriptions of particular figures, as Weber would have it.

Historians of Mathematics
This brings us to the last category in the scale built on the basic division between non-historical and historical postures towards the past. That “historians of mathematics” is a separate category and just not another subtype of the last is of course a matter of controversy. A central moment in the controversy was Sabetai Unguru’s 1975 paper, “On the Need to Rewrite the History of Greek Mathematics.” For if one accepts that there is complete continuity between the mathematics of the past and that of present, then it stands to reason that a mathematician will be best placed to understand and interpret the mathematics of the past. Viewed in that light, therefore, “mathematician-historians” would be “historians of mathematics,” par excellence. The problem goes right back to the initial division, what I referred to as the basic division: the non-historical versus the historical relationship to the past. I adopted Michael Oakeshott’s views to make this distinction, but it is also Oakeshott that puts his finger on the difficulty: “If the historical past be knowable, it must belong to the present world of experience; if it be unknowable, history is worse than futile, it is impossible” (Oakeshott, 1933, p.107). In any relationship with the past—as Oakeshott would agree—we dance with the present, and all the more so when it comes to mathematics of the past.

But what distinguishes the “mathematician historians” from the “historians of mathematics” is that this uneasy relation between present and past is understood as a difficulty: for them, the past is a problem. “Historians of mathematics” take as their working assumption, a kind of null-hypothesis, that there is a discontinuity between mathematical thought of the past and that of the present. Faced with a mathematical text, “historians of mathematics” do not try and coordinate the text with the mathematics of the present, but to set it off from the present; they try to make it not more familiar but rather more strange, more foreign. They cannot make the past into present experience (and in this regard Collingwood might have been too optimistic about the goal of history) but they can try to make the pastness of the text palpable and, accordingly, bring out its own identity.

Even here there are subtypes. One subtype is the “philosophical historian of mathematics.” This type, like all “historians of mathematics,” has a clear view of the problematic connection between past and present. But “philosophical historians” pursue their thorough and precise historical work against the background of a more general philosophical framework. The link with that framework can be stronger or weaker, but, in general, it can be said their work exemplifies and is to an extent driven by their philosophical outlook. The main example I have in mind is Jacob Klein. Thus Klein’s Greek Mathematical Thought and the Origin of Algebra (1968/1934-36) was at once one of the most probing books on Greek mathematics and its early modern transformation and at the same time an embodiment of ideas Klein learned from Husserl. In particular, it puts into action the idea that history and philosophy become united in the attempt to “reactivate sedimented meanings.” This Klein describes as follows:

This interlacement of original production and “sedimentation” of significance constitutes the true character of history. From that point of view there is only one legitimate form of history: the history of human thought. And the main problem of any historical research is precisely the disentanglement of all these strata of “sedimentation,” with the ultimate goal of reactivating the “original foundations,” i.e. of descending to the
true beginnings, to the “roots,” of any science and, consequently, of all prescientific conceptions of mankind as well. Moreover, a history of this kind is the only legitimate form of epistemology (Klein, 1985, p.78).

And Klein completes this passage by saying, “History, in this understanding, cannot be separated from philosophy,” which expresses precisely the differentia of this subtype.

Now “historical historians of mathematics,” which includes Sabetai, may share the philosophical outlook of a “philosophical historian of mathematics,” such as Jacob Klein, but differ in not having philosophy as their goal. They bear some similarity to “treasure hunters” in that they look for traces of lost mathematics. But for them it is not the mathematical work alone that has been lost; rather it is the very mathematical thought behind it. They aim to struggle with mathematical thought of the past as a kind of human thought recognizable somehow as mathematical but different than modern mathematical thought. Their main end, as I said above, is to make the pastness of past mathematical thought stand out in clear relief. As Sabetai himself has put it:

The historian of ideas does not discharge his obligation by showing merely the extent to which past ideas are like modern ideas. His main effort should be in the direction of showing the extent to which past ideas were unlike modern ones, irrespective of the fact that they might (or might not) have led to the modern ideas. This is a wise methodological tack, since it enables the historian to avoid reductive anachronism while channeling his historical empathy toward and understanding of the past in its own right. (Unguru, 1979, p.562).

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**Non-Historical**

**Mathematicians:**
- Colleagues
- Treasure hunters
- Conquerors

**Mathematician Historians:**
- Moderators
- Privileged observers
- Critics

**Historians of Mathematics:**
- Philosophical
- Historical

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**Educationalist Historians of Mathematics**

The types we have discussed to this point and their arrangement along the scale from the non-historical to historical extremes are represented in figure 1. But we must now take up the question that is really of greatest importance to all of us here, namely, is there a distinctively educational relationship to the past, is there an “educational historian of mathematics”? If so, where should we place that type? Can it be placed among those in the last category, the “historians of mathematics”? This is possible: there are streams in
mathematics education that look at cultural difference and are interested in showing different ways of thinking. This was the implicit position in Fried (2001, 2007). However, it is also possible and even likely that an “educational historian of mathematics” will be more at home among the “mathematician historians,” or even the “mathematical conquerors” engaged in showing how the mathematics of the past proves the importance of the standard modern mathematics taught in the classroom or “mathematical treasure hunters” finding gems from history forgotten in the mathematics curriculum.

It is not by chance that the educational historian of mathematics can be placed at almost every position along the scale in figure 1. Most of the figures we have discussed were active teachers in one way or another, and all were teachers in the sense that they wrote to communicate and enlighten. As already mentioned, Halley had to teach the classics of mathematics as part of his position as Savillian Professor of Geometry, and Jacob Klein shaped the program at St. John’s College where students were to read (and still do), Euclid, Apollonius, Newton, and Lobachevski. It is reasonable that that their particular relationship to history figured in their approach to teaching (and it certainly did in the case of Jacob Klein), and, conversely, how they thought historical texts could educate was directly related to their position in the non-historical-historical scale.

But the way teachers of mathematics teach mathematics is not only determined by their understanding of the nature of mathematics, historical or otherwise, but also by their own teaching goals and the kinds of problems they hope their teaching practices will solve. Relating to history in this context would again be an Oakeshottian “practical past,” a past to be used “as a tool,” as Uffe Jankvist (2009) has put it. In this sense, it seems to be located at the non-historical end of the scale I have discussed until now. Yet, the situation is not that simple, for one can treat history of mathematics as something to use but to use according to its specifically historical character. An educational relationship to the past might thus be located on a parallel axis to the one I have drawn (figure 2). With that possibility in mind, we can take Otto Toeplitz as an example of an “educationalist historian of mathematics.”

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**Figure 2: The parallel educationalist-historian axis**
Toeplitz’s book *Calculus: The Genetic Approach* (1963, originally published in German in 1949), which teaches the calculus according to its historical development. It is probably the best-known example of the approach of mathematics teaching according to the educational “ontogeny recapitulate phylogeny” framework (see Schubring, 2011; Fried, in press). Toeplitz’s rationale for this book, however, was set out not in the book itself but in an address at a congress held in Düsseldorf in 1926 (Toeplitz, 1927). The primacy of his educational focus is already evident in his title, “The problem of university infinitesimal calculus courses and their demarcation from infinitesimal calculus in high schools”—no mention whatsoever of history, even though his genetic approach is the centerpiece of the address. Toeplitz is trying to solve specifically educational problems. This is most clear in what he calls the “indirect genetic approach.” This, he says, consists of “…the elucidation of didactic difficulties, I should say, didactical diagnosis and therapy (didaktische Diagnose und Therapie), on the basis of historical analyses, where these [historical analyses] serve only to turn [one’s] attention in the right direction (Toeplitz, 1927, p.99, translation by Jahnke & Fried).” In fact, elsewhere in the address, Toeplitz states that the teaching itself does not have to refer to history explicitly. Toeplitz also speaks of a “direct genetic approach” that does bring history into the classroom directly and which is indeed what we see in Toeplitz’s calculus text; however, the indirect approach shows how history is an educational tool.

**Conclusion**

It is very easy to confound what I have tried to sketch here with a history of historiography. This is especially so in light of Zeuthen, Truesdell, Klein, and Unguru—since these figures all claim to be doing history, *per se*. But part of what I wanted to impress upon you is that those who claim to do history, *per se*, are only one group of those who relate to mathematics of the past in some way. Mathematicians working on mathematics will also have a particular relation to their predecessors. Mathematics educators also have a particular relation to the past both implicitly and explicitly. They have an unavoidable implicit relation because they teach mathematics at certain stage in the development of mathematics, the presuppositions about mathematics they bring to their teaching—what is interesting, what is useful, what is important—either continue or are set opposed to an older tradition. This kind of implicit relation is one shared by all those engaged in mathematics. But where mathematics educators bring history explicitly into their teaching, their relationship becomes a function of the kinds of didactic problems they want to solve. This is what we saw so clearly in Toeplitz. Those didactic considerations make this relationship different from the others. On the other hand, those didactic considerations include not only the very practical problems of the classroom, such as motivation, but also the question of what wants students to know finally. If one does not wish to dictate the latter to students, then, it will be important for students to come to terms with their own way of relating to the mathematical tradition. In this way, the educational mathematical historian may actually be the one who uses history to teach the ways in which one can relate to history, that is, to encourage cognizance of all these types, to keep them all alive.

**References**


